MATH 2010E Midterm, June 10, 2022
Time: 1:30 PM-2:45 PM
Submission due: 3:05 PM

- Make your solution into a single PDF file and upload on Blackboard by 3:05 PM.
- There are 6 problems on this exam, worth 60 points in total.
- If you have questions during the exam, please join the zoom meeting and send the lecturer a direct message(preferred) or an email.
- The Midterm Exam is open-book/notes. You are allowed to make use of the internet during the exam, except for the purpose of communicating with another individual besides the lecturer and TA of this course. Calculators are allowed.
- You are not allowed to post questions or answers on any internet forum or tutoring / educational establishment during the exam. Any violation of this rule is considered an act of academic misconduct, and will be reported to the Department for further disciplinary review.
- All work submitted should be your own. Students found to have highly similar answers could be subjected to disciplinary review.

Unless otherwise noted, please justify all your answers.

1. (10 pts) Let L_1 and L_2 be two lines in $\mathbb{R}^3$ given as follows. Find a plane containing both L_1 and L_2. If such a plane does not exist, explain why.

   (a) (5 pts) $L_1 : \dfrac{x-2}{1} = \dfrac{y+1}{2} = \dfrac{z-4}{3}$ and $L_2 : \dfrac{x-3}{2} = \dfrac{y}{4} = \dfrac{z-1}{6}$ in symmetric form.

   (b) (5 pts) $L_1 : (x, y, z) = (t, t, t)$ and $L_2 : (x, y, z) = (1+2s, -s, 7+3s)$ in parametric form ($t, s \in \mathbb{R}$).

2. (10 pts) Let $r(t)$ be a curve defined by $r(t) = (e^t, 2t, 2e^{-t}), t \in [0, 2]$.

   (a) (5 pts) Find a parametrization of the tangent line of $r(t)$ at $t = 1$.

   (b) (5 pts) Find the arclength of $r(t)$.

3. (10 pts) Let f be a function on $\mathbb{R}^2$ defined by

$$f(x, y) = \begin{cases} \dfrac{xy^m}{x^2 + 7y^8} & \text{if}(x, y) \neq (0, 0), \\ m & \text{if}(x, y) = (0, 0). \end{cases}$$

   Find all positive intgers m such that $\lim_{(x,y)\to(0,0)} f(x, y)$ exists.

4. (10 pts) Let f be a function on $\mathbb{R}^2$ defined by

$$f(x, y) = \begin{cases} \dfrac{x^2 y}{\sqrt{x^4 + x^2y^2 + y^4}} & \text{if } (x, y) \neq (0, 0), \\ 0 & \text{if } (x, y) = (0, 0). \end{cases}$$

   (a) (2 pts) Determine if f is continuous at $(0, 0)$.

   (b) (4 pts) Compute $f_x(0, 0)$ and $f_y(0, 0)$.

   (c) (4 pts) Determine if f is differentiable at $(0, 0)$.

5. (10 pts) Let $f = f(x, y)$ be a C^2-function of two variables defined on $\Omega \subset \mathbb{R}^2$. Let $\vec{v}$, $\vec{w}$ be two unit vectors in $\mathbb{R}^2$. Show that

$$D_{\vec{v}} (D_{\vec{w}} f) = D_{\vec{w}} (D_{\vec{v}} f)$$

   on Ω.

6. (10 pts) Suppose $f : \mathbb{R}^2 \to \mathbb{R}$ satisfies

$$|f(x, y) - (1 + 2y + x^2)| \leq \ln(1 + x^2 + y^2)$$

   for all $(x, y) \in \mathbb{R}^2$.

(a) (2 pts) What is $f(0,0)$?

(b) (3 pts) Show that $f(x,y)$ is continuous at $(0,0)$.

(c) (5 pts) Is f differentiable at $(0,0)$? If yes, prove and give the linear approximation of f at $(0,0)$. If no, give a conter-example.